Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Join BH. Take BO, OR, etc., equal to AB. Through O, R, etc., draw OM, RL, etc., parallel to BA, marking the lines of division MN, LR, etc.

Hence the square $ABCD$ forms the parts of the rectangle, AES, SM, MR, RLH, HGF.

Corollary. When AS is greater than AB, or conversely, when AB is less than AS, the construction is quite simple.

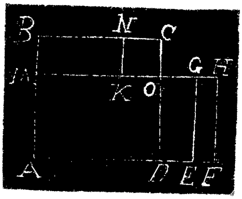
V. Solution by J. W. SCROGGS, Principal of Rogers Academy, Rogers, Arkansas.

Let $ABCD$ be the given square, $AB=a+b$, $AM=MK=OD=GE=a$, $OK=OC=DE=b$, and $EF=b/r$.

Then area of $ADOM=a^2+ab$, and area of $BNKM=ab$.

$\therefore$ Their sum$=a^2+2ab$.

Let $b/a=r$. Then $a=br$, and area of $EFHG=b/r\times br=b^2$. $\therefore$ $AFHM=ABCD$.



CALCULUS.

Conducted by J. M, COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

49. Proposed by B. F. BURLESON, Oneida Castle, New York.

Find (1) in the leaf of the strophoid whose axis is a the axis of an inscribed leaf of the lemniscata, the node of the former coinciding with the crunode of the latter. Find (2) in a leaf of the lemniscata whose axis b the axis of a of an inscribed leaf of the strophoid, the node of the former also coinciding with the crunode of the latter.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Case I.

The equation of strophoid with origin at node is,

$$y^2=\frac{x(x-a)^2}{2a-x}\text{.........................}(1).$$

The equation of lemniscate with origin at crunode is,

$$(x^2+y^2)^2=b^2(x^2-y^2)\text{... ....}(2).$$

In order that the latter may be inscribed in the former we must have tangency. $\therefore$ x, y, and dy/dx must be equal for each curve.

Substituting y from (1) in (2) we get after reduction,

$$x^3-4ax^2+\left(\frac{9a^2b^2-a^4}{2b^2}\right)x-a^3=0 \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

$$\frac{dy}{dx}=\frac{b^2x-2x^3-2xy^2}{y(b^2+2x^2+2y^2)}=\frac{4ax^2-4a^2x+a^3-x^3}{y(2a-x)^2}\ldots\ldots\ldots\ldots(4).$$

Substituting y from (1) in (4) and reducing we get,

$$x^4-6ax^3+12a^2x^2-\left(\frac{17a^3b^2-2a^5}{2b^2}\right)x+a^4=0\ldots\ldots\ldots\ldots(5).$$

(5) may be written as follows :

$$x^4-5ax^3+8a^2x^2-\left(\frac{8a^3b^2-a^5}{2b^2}\right)x-a\left\{x^3-4ax^2+\left(\frac{9a^2b^2-a^4}{2b^2}\right)x-a^3\right\}=0.$$

(3) in the last equation gives,

$$x^3-5ax^2+8a^2x-\frac{8a^3b^2-a^5}{2b^2}=0\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

(3)$-$(6) gives, $x^2-\dfrac{7ab^2+a^3}{2b^2}x-\dfrac{a^4-6a^2b^2}{2b^2}=0 \ldots\ldots\ldots\ldots\ldots\ldots(7).$

$\dfrac{8b^2-a^2}{2b^2}$ times (3)$-$(6) gives,

$$(6b^2-a^2)x^2-(22ab^2-4a^3)x+\frac{40a^2b^4-17a^4b^2+a^6}{2b^2}=0\ldots\ldots(8).$$

$(6b^2-a^2)$ times (7)$-$(8) gives,

$$x=\frac{4ab^4-5a^3b^2}{2b^4-7a^2b^2+a^4}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9).$$

$\dfrac{40a^2b^4-17a^4b^2+a^6}{2b^2}$ times (7)$+\dfrac{a^4-6a^2b^2}{2b^2}$ times (8) gives,

$$x=\frac{16ab^6+13a^3b^4-18a^5b^2+a^7}{8b^6-10a^2b^4}\ldots\ldots\ldots\ldots\ldots(10).$$

From (9) and (10) we get,

$$6b^8+161a^2b^6-141a^4b^4+25a^6b^2-a^8=0\ldots\ldots\ldots\ldots(11).$$

Let $a^2/b^2=u$, then (11) becomes,

$$u^4-25u^3+141u^2-161u-6=0\ldots\ldots\ldots\ldots(12).$$

$\therefore\ u=1.586892.\quad \therefore\ a^2=1.586892b^2.$

$\therefore\ a=1.2597b.\quad \therefore\ b=.7938a.$

Case II.

The equation of the strophoid with origin at crunode is,

$$y^2=\frac{x^2(a+x)}{a-x}\ldots\ldots\ldots\ldots(13).$$

The equation of the lemniscate with origin at node is,

$$\{(x+b)^2+y^2\}^2=b^2\{(x+b)^2-y^2\}\ldots\ldots\ldots\ldots(14).$$

In order that the former may be inscribed in the latter we must have x, y, dy/dx equal for both curves.

(13) in (14) gives,

$$(2a^2+b^2-4ab)x^3+(4a^2b-5ab^2+b^3)x^2+(4a^2b^2-2ab^3)x+a^2b^3=0\ldots\ldots(15).$$

$$\frac{dy}{dx}=\frac{a^2x+ax^2-x^3}{y(a-x)^2}=-\frac{2x^3+6bx^2+5b^2x+b^3+2(x+b)y^2}{y(2x^2+4bx+3b^2+2y^2)}\ldots\ldots(16).$$

(13) in (16) gives,

$$(8ab-4a^2-2b^2)x^4+(8a^3-20a^2b+9ab^2-b^3)x^3$$
$$+(12a^3b-15a^2b^2+3ab^3)x^2+(8a^3b^2-3a^2b^3)x+a^3b^3=0\ldots(17).$$

(17) may be written

$$(8ab-4a^2-2b^2)x^4+(6a^3-16a^2b+8ab^2-b^3)x^3$$
$$+(8a^3b-10a^2b^2+2ab^3)x^2+(4a^3b^2-a^2b^3)x$$
$$+a[(2a^2+b^2-4ab)x^3+(4a^2b-5ab^2+b^3)x^2+(4a^2b^2-2ab^3)x+a^2b^3]=0\ldots(18).$$

(15) in (18) gives,

$$2(2a^2+b^2-4ab)x^3+(16a^2b-8ab^2+b^3-6a^3)x^2$$
$$+(10a^2b^2-8a^3b-2ab^3)x+a^2b^3-4a^3b^2=0\ldots(19).$$

(19)−2 times (15) gives,

$$(8a^2b+2ab^2-b^3-6a^3)x^2+(2a^2b^2-8a^3b+2ab^3)x-a^2b^3-4a^3b^2=0 \quad \ldots\ldots(20).$$

b times (19)−$(b-4a)$ times (15) gives,

$$(8a^3+b^3-14a^2b)x^2+(ab^3+10a^3b-8a^2b^2)x+8a^3b^2-2a^2b^3=0 \quad \ldots\ldots\ldots\ldots(21).$$

$(8a^3+b^3-14a^2b)$ times (20)−$(8a^2b+2ab^2-b^3-6a^3)$ times (21) gives,

$$x=\frac{14a^3b^3+16a^5b+8a^2b^4-28a^4b^2-3ab^5}{4a^5-38a^3b^2+8ab^4+18a^2b^3-3b^5} \quad \ldots\ldots\ldots\ldots(22).$$

$(8a-2b)$ times (20)+$(4a+b)$ times (21) gives,

$$x=\frac{10a^3b^2-24a^4b+8a^2b^3-3ab^4}{14a^2b^2+16a^4+8ab^3-28a^3b-3b^4} \quad \ldots\ldots\ldots\ldots\ldots(23).$$

From (22) and (23) we get,

$$9b^5+16ab^4-148a^2b^3-144a^3b^2+468a^4b-176a^5=0\ldots\ldots\ldots(24).$$

Let $b/a=v$, then (24) becomes,

$$9v^5+16v^4-148v^3-144v^2+468v-176=0.$$

$\therefore\ v=1.12257.\quad \therefore\ b=1.12257a.\quad \therefore\ a=.8908b.$

The published solution assumes that both curves coincide at their crunodes. This is not what the problem calls for. The above solution realizes in every respect the demands of the problem.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

44. Proposed by O. W. ANTHONY, M. Sc., Columbian University, Washington, D. C.

There is a triangle whose sides repulse a center of force within the triangle with an intensity that varies inversely as the distance of the center of force from each point of the sides of the triangle. What is the position of equilibrium of the center?